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A Solution of One-dimensional Advection-diffusion equation for Concentration Distribution in fluid flow through Porous Media by Homotopy Analysis Method

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ABSTRACT
The governing equation of longitudinal dispersion phenomenon of one-dimensional concentration distribution in fluid flow through porous media has been obtained in term of one-dimensional non-linear advection-diffusion equation. This equation has been converted in term of dimensionless non-linear Burger's equation with its derivative, and it is multiplied by small parameter $\varepsilon \in [0,1]$. This equation has been solved using Homotopy analysis method with appropriate initial and boundary condition and it is concluded that the concentration distribution of miscible fluids (i.e. contaminated or salt water with fresh water) decreases for given value of X and T > 0. the graphical and numerical presentation is derived using Maple coding.
Keywords - Advection, Burger's equation, Diffusion, Homtopy analysis method

I. INTRODUCTION

Groundwater and solute transport in coastal subsurface environments have significant implications for studying physical, chemical and biological processes in coastal area. The problem of solute dispersion during ground water movement has attracted interest from the early days of this century [1], but it was only since 1905 in general topic of hydrodynamic dispersion or miscible displacement becomes one of the more systematic studies. The phenomenon of the dispersion has been receiving good attention from hydrologist, agriculture, environmental, mathematicians, chemical engineering and soil scientists. The specific problem of fluid mixing in fixed bed reactors has been investigated by Bernard and Wilhelm [2]. Kovo [3] has worked with the parameter to be modeled in the longitudinal or axial dispersion coefficient D in chemical reactors model. In ground water hydrology intrusion of sea water into the coastal aquifers is an example of hydrodynamic dispersion. Saltwater intrusion as shown in figure 1 occurs where too much freshwater is pumped out of the ground and is replaced by brackish and eventually saltwater. The phenomenon of miscible displacement can be observed in coastal areas, where the fresh water is gradually displaced by sea water and a transition zone develops between fresh water and sea water. The transition zone between salt and fresh water is often quite narrow in comparison with the overall thickness of the aquifer and for computational purpose we may consider it to be a sharp interface [4], such a sharp interface approximation serve as justification for treating saltwater intrusion into coastal aquifers as a multiphase flow.

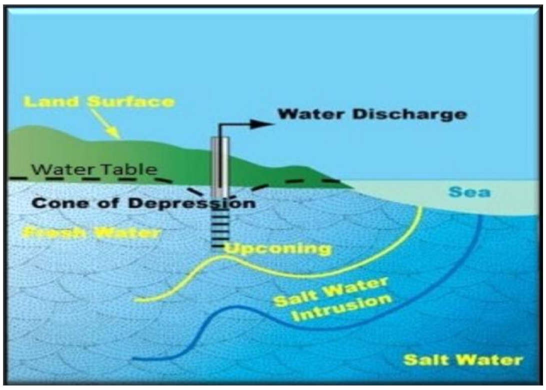

Figure 1: Coastal area applications: Salt water intrusion

In the case of salt water intrusion, over pumping from an aquifer creates a flow imbalance within an area, this results in salt water intruding into and polluting a fresh water supply. Both of the phases are of different salinity. Across this zone the concentration of water varies gradually from that of fresh water to that of sea water. This type of problem of salinity is shown at Saurashtra area of Gujarat due to the intrusion of seawater in the coastal aquifers and seawater ingress. At the same time, rising groundwater levels in the command area of Mahi irrigation scheme cause soil salinity problems in South Gujarat (India) [5]. Another example is when water of one quality is introduced into an aquifer containing water of another quality by various artificial recharge methods (surface spreading techniques or injection through wells). Presently efforts are being made by the environmentalists to dispose the atomic waste products, born from nuclear reactors, and dump it inside the ground by using the same phenomenon of displacement.

The fundamental interest of this paper is to find concentration of contaminated or salt water in soil. The term concentration expresses a measure of the amount of a substance within a mixture. The processes concentration distribution can be divided into two parts: transformation and transport.

1. Transformation: It refers to those processes that change a substance of interest into another substance. The two basic approach of transformation are physical (e.g. radioactive decay) and chemical reactions (e.g. dissolution).

2. Transport: It refers to those processes which move substance through the hydrosphere and atmosphere by physical means i.e. a substance goes from one location to another.

The dispersion process is associated with molecular diffusion and mechanical dispersion. Molecular diffusion is the spreading caused by the random molecular motion and collisions of the particles themselves and mechanical dispersion is the spreading of a dissolved component in the water phase by variations in the water velocity (i.e. flow of a fluid). These two basic mechanisms molecular diffusion and mechanical dispersion cause a concentration front of fluid particles to spread as it advances through the porous media. These two combine processes of molecular diffusion and mechanical dispersion are known as hydrodynamic dispersion or dispersion.

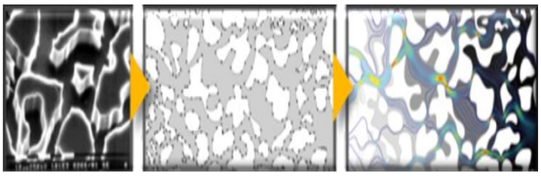

Fig. 2: The geometry of microscopic pores, where velocity distributions in different pore size.

When groundwater flows, the actual microscopic velocity in the pores varies widely in space even when the Darcy macroscopic velocity is constant. The result is more intense mixing, which is called hydrodynamic dispersion. Fig. 2 gives a schematic view of the trace movement on macroscopic level. This phenomenon can be observed in coastal areas, where seawater gradually displaced the fresh waterbeds. An important role is played by this phenomenon in the seawater intrusion into reservoir at river mouths and in the underground recharge of groundwater.

Several investigators have evaluated the dependence of the dispersion parameter D on the velocity u that appears in one -dimensional advection – diffusion equation.

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left\{ D\frac{\partial C}{\partial x} - u(x,t)C \right\}$$

In this analysis, Taylor [6] found dispersion coefficient D proportional to square of uniform velocity (u^2); Bears and Todd [7] suggests that D is directly proportional to u; Scheiegger [8] in his study of the possible relationships summarizes: firstly, $D = \alpha u^2$, where α, the porous medium constant, is derived by a dynamic procedure valid when there is enough time in each flow channel for appreciable mixing to take place by molecular transverse diffusion. Secondly, $D = \beta u$, where β is another constant of the porous medium, that is derived by a geometrical procedure relevant where there is no appreciable molecular transverse diffusion from one streamline into another. In light of these, the dispersion problem has been made two-dimensional considering transverse diffusion. Freeze and Cherry [9] advocates, the dispersion parameter is proportionate to the power n of the velocity; where the range of power arrays between 1 and 2. Most of the works reveal common assumption of homogeneous porous media with constant porosity, steady seepage flow velocity and constant dispersion coefficient. For such assumption, Ebach and White [10] studied the longitudinal dispersion problem for an input concentration that varies periodically with time. Hunt [11] applied the perturbation method to longitudinal and lateral dispersion in no uniform seepage flow through heterogeneous aquifers. Saffman and Taylor [12] showed that if dispersion is mainly due to convective mixing (kinematic and dynamic dispersion process) but there is small but finite effect of molecular diffusion, the dispersion coefficients (longitudinal and transverse) are related to the mean flow velocity. Rudaria and Chiu-On Ng [13] has provided analytical study of the dispersion in fluid-saturated deformable or non deformable porous media with or without chemical reaction, considering a series of particular cases selected through different practical problems using different dispersion models. Mehta and Patel [14] applied Hope-Cole transformation to unsteady flow against dispersion of miscible fluid flow through porous media. Eneman et al. [15] provided analysis for the systems where fresh water is overlain by water with a higher density in coastal delta areas. Meher [16] and Mehta [17] studied the Dispersion of Miscible fluid in semi infinite porous media with unsteady velocity distribution using Adomain decomposing method. Moreira et al (2006, 2009) [18], [19] presented an analytical solution for the non-stationary two-dimensional advection-diffusion equation to simulate the pollutant dispersion in the planetary boundary layer. They solved the advection-diffusion equation by the application of the Laplace transform technique and the solution of the resulting stationary problem by the generalized integral Laplace transform technique (GILTT). For temporally and spatially dependent dispersion problems, the analytical solutions were obtained by Jaiswal et al (2009, 2011) [20-21], Yadav et al. [22] has obtained solution for one-dimensional advection-diffusion equation with variable coefficients in a longitudinal domain. It was demonstrated by Dong-mei et al. [23] that two-phase flow concept is the basis for unsteady seepage velocity that influences an infiltration of the rainfall and water level variation with seepage flow. An exact solution of the linear advection-dispersion transport equation with constant coefficients was introduced by Perez et al. [24] for both transient and steady state regimes and classic version of Generalized Integral Transform Technique (GITT) was used in solving analytically. Three simple time-dependent inlets conditions comprising regularly, rapidly declining and sinusoidally periodic input functions; were used to demonstrate the applicability of the solution by Chen and Liu [25] while studying a broader analytical solution for one-dimensional advection-dispersive transport infinite spatial domain.

The present paper discusses the approximate analytical solution of the nonlinear differential equation for longitudinal dispersion phenomenon which takes places when miscible fluids (contaminated or salt water) mix in the direction of flow. The mathematical formulation of the problem yields a nonlinear second order parabolic partial differential equation. The analytical solution has been obtained by using Homotopy analysis method. The graphical representation along with its physical interpretation is also discussed.

the introduction of the paper should explain the nature of the problem, previous work, purpose, and the contribution of the paper. The contents of each section may be provided to understand easily about the paper.

II. Statement of the problem

Considering dispersion of contaminated or salt water with concentration $C(x,t)$ flowing in x-direction, dispersion taking place in porous media saturated with fresh water. Hence it will be miscible fluid flow through homogeneous porous media. Therefore, it will obey the Darcy's law, which dates back to 1856 [1]. The following assumptions have been made for present analysis (Schidegger 1954, Day 1956, deJony 1958) [26, 27, 28]:

- The medium is homogenous.
- The solute transport across any fixed plane,

due to microscopic velocity variation in the flow tube, may be quantitatively expressed as the product of a dispersion coefficient and the concentration gradient.

To find concentration of the dispersing contaminated or salt water as a function of time t and distance x, as the two miscible fluids flow through homogeneous porous media. Since the mixing (contaminated or salt water and fresh water) takes place both longitudinally and transversely. Dispersion adds a spreading effect to the diffusion effects. Since dispersion is driven by the mean flow of the water, the dispersion coefficients related to the characteristic length or pore length L. In three dimensions, the spreading caused by dispersion is greater in the direction of the flow than in the transverse direction. One dimensional treatment of these systems avoids treatment of a radial or transverse component of dispersion. We only consider the dispersion phenomenon in the direction of flow (i.e. longitudinal dispersion), which takes places when miscible fluids flow in homogeneous porous media.

III. Mathematical Structure

The dispersion equation that describes the concentration distribution of miscible fluids (i.e. contaminated or salt water with fresh water) flow in homogeneous porous media can be written as (Freeze and Cherry [9] and Bear [29]),

$$\frac{\partial C}{\partial t}+\frac{1-\phi}{\phi}\frac{\partial F}{\partial t}=\frac{\partial}{\partial x}\left\{D\frac{\partial C}{\partial x}-u(x,t)C\right\} \tag{1}$$

where C is the concentration of dispersing contaminated or salt water, F is the concentration in the solid phase, ϕ is porosity of medium, u is seepage velocity of contaminated or salt water and D is the dispersion coefficient presenting, at the macroscopic scale, which presumably includes the effect of both molecular diffusion and mixing in the axial direction, however molecular diffusion is negligible due to very low seepage velocity. In equation (1) dispersion coefficient D and seepage velocity u may be constant or functions of x and t and $D>0$ [30].

Let u_x is the component of seepage velocity of contaminated or salt water along the x axis, then the non-zero components will be $D_{11} = D_L = \frac{L}{C_0^2}$, (coefficient of longitudinal dispersion and L is length of dispersion in flow direction) and $D_{22} = D_T$ (coefficient of transverse dispersion) and other D_{ij} are zero. From this assumption the equation (1) becomes,

$$\frac{\partial C}{\partial t} + \frac{1-\phi}{\phi}\frac{\partial F}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - u_x \frac{\partial C}{\partial x} \tag{2}$$

Lapidus and Amundson [30] considered two cases as,

$$F = K_1 C + K_2 \tag{3}$$

$$\frac{\partial F}{\partial t} = K_1 C - K_2 F \tag{4}$$

for equilibrium and non-equilibrium relationship between the concentrations in the two phases, respectively. Using equation (3) in equation (2) can be written as,

$$\frac{\partial C}{\partial t} + \frac{1-\phi}{\phi}\frac{\partial (K_1 C + K_2)}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - u_x \frac{\partial C}{\partial x}$$

or $$R\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - u_x \frac{\partial C}{\partial x} \tag{5}$$

where $R = \left(1 + \frac{1-\phi}{\phi}K_1\right)$ is a retardation factor describing solute sorption, K_1 and K_2 are empirical constants, u_x is the component of seepage velocity of contaminated or salt water in x direction which is function of x and t and $D_L > 0$. It has been observed that the component of seepage velocity u_x (along with the x axis) is related with concentration of contaminated or salt water dispersion. We assume that seepage velocity u_x is directly proportional to $C(x,t)$ [17].

$$u_x = \frac{C(x,t)}{C_0} \tag{6}$$

where $1/C_0$ is constant of proportionality and the guess approximation of the concentration of contaminated or salt water dispersion. Following new independent variables has been introduced to simplify the equation (5) as,

$$X = \frac{C_0 x}{L}, and\ T = \frac{R}{L}t$$

then equation (5) can be written as,

$$\frac{\partial C}{\partial T} = -C\frac{\partial C}{\partial X} + \varepsilon \frac{\partial^2 C}{\partial X^2} \tag{7}$$

where $\varepsilon = \frac{D_L C_0^2}{L}$ and $\varepsilon \in [0,1], 0 \le X \le 1, 0 \le T \le 1$

Since concentration C is decreasing as distance X increase for T > 0. It appropriate to choose guess value of concentration solution as, [16]

$$\mathbb{C}(X,T;\varepsilon) = (1-T)e^{-X} + \varepsilon^m \tag{8}$$

Hence, the equation (7) together with boundary condition (8) represents the governing non-linear partial differential equation for concentration of the longitudinal dispersing material of miscible fluids flowing through a homogeneous porous medium.

IV. THE SOLUTION WITH HOMOTOPY ANALYSIS METHOD

For one dimensional non-linear partial differential equation for longitudinal dispersion phenomenon, we assumed that the concentration $C(X,T)$ of the dispersing contaminated or salt water, at time T=0 is expressed as,

$$\mathbb{C}(X,T,\varepsilon) = (1-T)e^{-X} + \varepsilon^m \tag{9}$$

where $\varepsilon = 0$ for the concentration of contaminated or salt water for time T = 0.

Now we apply the Homotopy analysis method into the longitudinal dispersion phenomenon during miscible fluid flow through homogeneous porous media. We consider the equation (7) as nonlinear partial differential equation as

$$\mathbb{N}[\mathbb{C}(X,T;\varepsilon)] = 0 \tag{10}$$

Where $\mathbb{N}$ is a non-linear operator, $\mathbb{C}(X,T;\varepsilon)$ is considered as unknown function which represent the concentration C of the dispersing contaminated or salt water at any distance X for given time $T > 0$, for $0 \le \varepsilon \le 1$. We use auxiliary linear operator $\Im[\mathbb{C}(X,T;\varepsilon)] = \frac{\partial \mathbb{C}(X,T;\varepsilon)}{\partial T}$ and initial approximation of concentration of dispersing contaminated or salt water $C_0(X,T) = (1-T)e^{-X}$ to construct the corresponding zero^th order deformation equation. As the auxiliary linear operator $\Im$ which satisfies $\Im[C_4] = 0$, where C_4 is arbitrary constant. This provides a fundamental rule to direct the choice of the auxiliary function $H(X,T) \ne 0$, the initial approximation $C_0(X,T)$, and the auxiliary linear operator $\Im$, called the rule of solution expression. Establish the zero-order deformation equation of longitudinal dispersion phenomenon as [31],

$$(1-\varepsilon)\Im[\mathbb{C}(X,T;\varepsilon) - C_0(X,T)] = \varepsilon\hbar H(X,T)\mathbb{N}[\mathbb{C}(X,T;\varepsilon)] \tag{11}$$

where $C_0(X,T)$ denote an initial guess value of concentration of dispersing contaminated or salt water of the exact solution $C(X,T)$ which is our purpose to find it. Since $\hbar \ne 0$ is an auxiliary parameter and $H(X,T) \ne 0$ is an auxiliary function such that $\varepsilon \in [0,1]$ is an embedding parameter. The auxiliary parameter $\hbar$ is providing a simple way to ensure the convergence of series. Thus it renamed $\hbar$ as convergence control parameter [31]. Let $\Im$ an auxiliary linear operator with the property that,

$\Im\left[\mathbb{C}(X,T;\varepsilon)\right]=0$ when $C(X,T;\varepsilon)=0$

when $\varepsilon=0$, the zero-order deformation equation (11) becomes

$$\Im\left[\mathbb{C}(X,T;\varepsilon)-C_0(X,T)\right]=0 \tag{12}$$

Which gives the first rule of solution expression and according to the initial guess $C_0(X,T)=(1-T)e^{-X}$, it is straightforward to choose

$$\mathbb{C}(X,T;0)=C_0(X,T) \tag{13}$$

when $\varepsilon=1$, since $\hbar\neq 0$, $H(X,T)\neq 0$ the zero-order deformation equation (7) is equivalent to

$$\mathbb{N}\left[\mathbb{C}(X,T;\varepsilon)\right]=0 \tag{14}$$

which is exactly the same as the original equation (10) provided

$$\mathbb{C}(X,T;1)=C(X,T) \tag{15}$$

According to (13) and (15) as the embedding parameter ε increases from 0 to 1, solution $\mathbb{C}(X,T;\varepsilon)$ varies continuously from the initial guess value of the concentration $C_0(X,T)$ of dispersing contaminated or salt water to the solution $C(X,T)$ and its solution is assumed by expanding $\mathbb{C}(X,T;\varepsilon)$ in Taylor series with respect to ε as,

$$\mathbb{C}(X,T;\varepsilon)=\mathbb{C}(X,T;0)+\sum_{m=1}^{\infty}C_m(X,T)\varepsilon^m \tag{16}$$

Where, $$C_m(X,T)=\frac{1}{m!}\left.\frac{\partial^m\mathbb{C}(X,T;\varepsilon)}{\partial\varepsilon^m}\right|_{\varepsilon=0} \tag{17}$$

i.e. the concentration of dispersing contaminated or salt water is function of distance X and time T for any parametric value ε is expressed as, the concentration of dispersing contaminated or salt water at time $T=0$, $C_0(X,T)$ and sum of concentration of dispersing contaminated or salt water $C_1(X,T)$, $C_2(X,T)$,…at different time T for different value of parameter ε. Here, the series (16) is called Homotopy-series; the series (16) is called Homotopy series solution of $\mathbb{N}\left[\mathbb{C}(X,T;\varepsilon)\right]=0$ and $C_m(X,T)$ is called the m[th]-order derivative of $\mathbb{C}$. Auxiliary parameter $\hbar$ in Homotopy-series (16) can be regard as iteration factor and is widely used in numerical computations. It is well known that the properly chosen iteration factor can ensure the convergence of Homotopy series (16) is depending upon the value of $\hbar$, one can ensure that convergent of Homotopy series, solution simply by means of choosing a proper value of $\hbar$ as shown by Liao [31, 32,33, 34]. If the auxiliary linear operator, the initial guess, the auxiliary parameter $\hbar$, the auxiliary function $H(X,T)$ are so properly chosen, the series (16) converges at $\varepsilon=1$.

Hence the concentration of dispersing water can be expressed as,

$$C(X,T)=C_0(X,T)+\sum_{m=1}^{\infty}C_m(X,T) \tag{18}$$

This must be one of solution of original non-linear partial differential equation (7) of the concentration of dispersing contaminated or salt water problem in homogeneous porous medium.

According to the definition (17), the governing equation can be deduced from the zero-order deformation equation (11), define the vector

$\vec{C}_m=\{C_0(X,T),C_1(X,T),...C_n(X,T)\}$

Differentiating equation (11) m-times with respect to the embedding parameter ε and then setting $\varepsilon=0$ and finally dividing them by $m!$, we have the so-called m[th] order deformation equation of the concentration $C(X,T)$ will be as, [31]

$$\Im\left[C_m(X,T)-\chi_m C_{m-1}(X,T)\right]=\varepsilon\hbar H(X,T)R_m\left(\vec{C}_{m-1},X,T\right) \tag{19}$$

Where

$$R_m\left(\vec{C}_{m-1},X,T\right)=\frac{1}{(m-1)!}\left.\frac{\partial^{m-1}\mathbb{N}\left[\mathbb{C}(X,T;\varepsilon)\right]}{\partial\varepsilon^{m-1}}\right|_{\varepsilon=0} \tag{20}$$

And $$\chi_m=\begin{cases}0, & m\le 1\\ 1, & m>1\end{cases} \tag{21}$$

It should be emphasized that $C_m(X,T)$ for $m\ge 1$, is governed by the linear equation (20) with the linear boundary condition that came from original problem, which can solved by symbolic computation software Maple as bellow. The rule of solution expression as given by equation (8) and equation (11), the auxiliary function independent of ε can be chosen as $H(X,T)=1$ [31].

According to (15) and taking inverse of equation (19) the equation (20) becomes,

$$C_m(X,T)=\chi_m C_{m-1}(X,T)+\hbar\Im^{-1}\left[R_m\left(\vec{C}_{m-1},X,T\right)\right] \tag{22}$$

$$R_m\left(\vec{C}_{m-1},X,T\right)=\frac{1}{(m-1)!}\left.\frac{\partial^{m-1}\mathbb{N}\left[\mathbb{C}(X,T;\varepsilon)\right]}{\partial\varepsilon^{m}}\right|_{\varepsilon=0} \tag{23}$$

In this way, we get $C_m(X,T)$ for m=1, 2, 3, … successively by using Maple software as,

$$C_1(X,T)=\frac{1}{6}\hbar T\left(-2T^2+6T+3Te^X-12e^X-6\right)e^{-2X} \tag{24}$$

$$C_2(X,T)=\frac{1}{60}T\hbar\begin{pmatrix}-12T^4\hbar+35T^3\hbar e^X+60T^3\hbar-120T^2\hbar\\ -10T^2\hbar e^{2X}-200T^2\hbar e^X+90T\hbar+300T\hbar e^X\\ +90T\hbar e^{2X}-120\hbar e^{2X}-60\hbar e^X-120e^{2X}\\ -20T^2e^X+60Te^X+30Te^{2X}-60e^X\end{pmatrix}e^{-3X} \tag{25}$$

....

Using initial guess value of concentration from equation (8) and successive

$$C(X,T)=(1-T)e^{-X}+\frac{1}{6}\hbar T\left(-2T^2+6T+3Te^X-12e^X-6\right)e^{-2X}$$
$$+\frac{1}{120}T\hbar\begin{pmatrix}-12T^4\hbar+35T^3\hbar e^X+60T^3\hbar-120T^2\hbar\\ -10T^2\hbar e^{2X}-200T^2\hbar e^X+90T\hbar+300T\hbar e^X\\ +90T\hbar e^{2X}-120\hbar e^{2X}-60\hbar e^X-120e^{2X}\\ -20T^2e^X+60Te^X+30Te^{2X}-60e^X\end{pmatrix}e^{-3X}+\ldots \tag{26}$$

V. NUMERICAL AND GRAPHICAL SOLUTION

Maple coding has been used to obtain numerical and graphical presentations of equation (26). Fig. 3 represents the graphs of concentration $C(X,T)$ vs. distance X, and time T = 0.1, 0.2, 0.3, 0.4, 0.5 is fixed, fig. 4 represents the graphs of concentration $C(X,T)$ vs. distance X, and time T = 0.6, 0.7, 0.8, 0.9, 1.0 is fixed and Table I indicates the numerical values of concentration for different time T and distance X. The fig. 3 & 4 and the table 1, indicate the graphical representations of the longitudinal dispersion phenomenon of the concentration. The convergence of the Homotopy series (16) is dependent upon the value of convergence-parameter $\hbar$ [31, 32, 33, 34]. Therefore we choose proper value of the convergence-parameter $\hbar=-0.1$ to obtain convergent Homotopy-series solution [31].

Table I : Concentration of the contaminated or salt water $C(X,T)$

Time T	Concentration of the contaminated or salt water									
	X=0.1	X=0.2	X=0.3	X=0.4	X=0.5	X=0.6	X=0.7	X=0.8	X=0.9	X=1.0
0.1	0.8555	0.7729	0.6986	0.6314	0.5707	0.5159	0.4664	0.4217	0.3813	0.3448
0.2	0.8029	0.7246	0.6539	0.5904	0.5332	0.4815	0.4349	0.3929	0.3551	0.3209
0.3	0.7476	0.6736	0.6072	0.5476	0.4939	0.4457	0.4023	0.3632	0.3279	0.2962
0.4	0.6893	0.6202	0.5584	0.5029	0.4532	0.4085	0.3684	0.3323	0.2999	0.2707
0.5	0.6282	0.5644	0.5074	0.4565	0.4109	0.3689	0.3334	0.3005	0.2709	0.2443
0.6	0.5646	0.5064	0.4546	0.4084	0.3671	0.3302	0.2972	0.2676	0.2411	0.2173
0.7	0.4986	0.4463	0.3999	0.3587	0.3219	0.2892	0.2589	0.2339	0.2105	0.1895
0.8	0.4303	0.3842	0.3436	0.3075	0.2755	0.2471	0.2218	0.1992	0.1789	0.1609
0.9	0.3599	0.3204	0.2856	0.2549	0.2278	0.2039	0.1826	0.1637	0.1469	0.1319
1.0	0.2876	0.2548	0.2262	0.2011	0.1789	0.1596	0.1425	0.1274	0.1139	0.1021

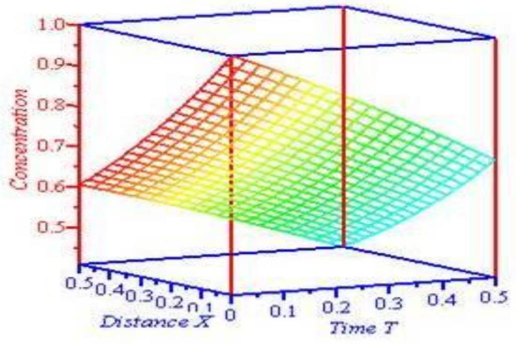

Figure 3: Represents concentration of contaminated or salt water $C(X,T)$ vs. distance X and time T for auxiliary parameter $\hbar=-0.1$ and auxiliary function $H(Z,T)=1$[31] for $0<X\le 0.5, and\ 0<T\le 0.5$

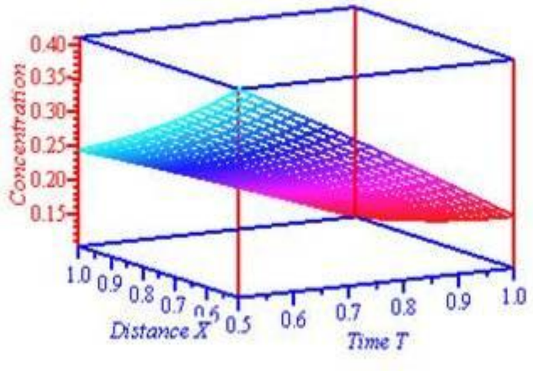

Fig 4: Represents concentration of contaminated or salt water $C(X,T)$ vs. distance X and time T for auxiliary parameter $\hbar=-0.1$ and auxiliary function $H(Z,T)=1$[31] for $0.5<X\le 1, and\ 0.5<T\le 1$

VI. CONCLUSION AND DISCUSSION

The equation (26) represents concentration of the miscible fluid (i.e. contaminated or salt water with fresh water) for any distance X and time $T>0$ using Homotopy Analysis Method. It converges for embedding parameter $\varepsilon=1$ and for auxiliary parameter $\hbar=-0.1$ which is expressed as negative exponential term of X and time $T>0$. An assumed value of concentration C will be a value of the exact solution for $0<X<1$,and $0<T<1$. Fig. 3 represents the solution for concentration C vs. distance X and time T for given X= 0.1, 0.2, 0.3, 0.4 and 0.5 fixed, it shows that concentration of the contaminated or salt water is decreasing as distance X increasing for $T>0$. From fig. 3 it can conclude that for $T=0.1$ concentration of contaminated or salt water is decreasing as distance X increasing and when time is increasing and due to different deformation added to C, the concentration of contaminated or salt water is successively decreasing exponentially. Since the equation (7) is one-dimensional diffusion type Burger's equation for longitudinal dispersion phenomenon, the solution is graphically as well as physically consistent with phenomenon. Fig. 4 represents the solution for concentration C vs. distance X and time T for given X= 0.5, 0.6, 0.7, 0.8, 0.9, 1.0. The concentration of contaminated or salt water is also decreasing for different time T for given fix value of X. The concentration of contaminated or salt water at $X=0.1$ is decreasing for different time T. After distance X, the concentration of contaminated or salt water is also decreasing with respect to different time T, this resembles the scenario of X above. Referring both fig. 3 and 4, with derived analytical result (26), it is concluded that the concentration of the contaminated or salt water is decreasing when distance X as well as time increases using Homotopy Analysis Method. This mathematical model is consistent with physical phenomenon of the longitudinal dispersion of contaminated or salt water in homogenous porous medium.

REFERENCES

[1] Darcy H., Les Fontaines publiques deloville *de Dijon, Dalmont, Paris, 1856*.
[2] Bernard, R. A., and R. H. Wilhelm, *Chem.Eng. Progr., 46, 1950, 233.*
[3] Kovo S.A., Mathematical modelling And simulation of dispersion in a nonideal plug flow reactor, *Taylor & Francis Group, 29, 2008, 1129-1134.*
[4] Pinder, G. F. and Cooper, H. H. Jr, A numerical technique for calculating the transient position of saltwater front, *water Resources Research, 6, 1970, 875-882.*
[5] Kumar M. D., Irrigation with a Manual Pump: Impact of Treadle pumps on Farming Enterprise and Food security in coastal Orissa. *Working paper 148. Anand: Insitute of Rural Management, 2002.*
[6] Taylor G., Dispersion of soluble matter in solvent flowing slowly through a tube, *Proc. R. Soc. London, series A, 219 (1137), 1953, 186-203.*
[7] Bear J., and Todd D. K., The transition zone between fresh and saltwater in coastal aquifers, *Water Resources Contribution No. 29, Univ. of California, Berkeley, CA, 1960*.
[8] Scheidgger A. E. The Physics of flow through porous media, *University of Toronto Press, Toronto, 1957.*
[9] Freeze R. A. and Cherry J. A., Groundwater, Prentice-Hall, *Upper saddle River, NJ, 1979.*
[10] Ebach E. H., and White, R. Mixing of fluids flowing through beds of packed solids, *A. J. Ch. E., 4, 1958, 161.*
[11] Hunt B., Dispersion calculations in nonuniform seepage, *J. Hydrol., 36, 1978, 261-277.*
[12] Saffman P. G. and Tylor G. I., The penetration of a fluid into a porous medium or Hele-Shaw cell containing a more viscous liquid. *Proc. R.Soc. London Series A, 245, 1959, 312-329.*
[13] Rudraiah N. and Chiu-On Ng., Dispersion in Porous Media with and without Reaction: *A Review. J. of Porous Media, 3, 2007.*
[14] Mehta M.N. & Patel, T. A., A Solution of Burger's Equation for Longitudinal Dispersion of Miscible Fluid Flow through Porous Media, *Indian Journal of Petroleum Geology, 14(2), 2005, 49-54.*
[15] Eeman S., Van Der Zee, S. E. A. T. M., Leijnse A., De Louw, P. G. B and Maas C., Response to recharge variation of thin lenses and their mixing zone with underlying saline groundwater Hydrol. *Earth Syst. Sci. Discuss., 16, 2012, 355-349.*
[16] Meher R. K. Mehta M. N. Meher S. K., Adomian Decomposition Method for Dispersion Phenomena Arising in Longitudinal Dispersion of Miscible Fluid Flow through Porous Media, *Adv. Theor. Appl. Mech., 3(5), 2010, 211-220.*
[17] Mehta M. N., Asingular perturbation solution of one-dimensional flow in unsaturated porous media with small diffusivity coefficient, *Proc. FMFP E1 to E4, 1975*.
[18] Moreira D. M., Vilhena M. T., Buske D., and Tirabassi T., The state-of-art of the ILTT method to simulate pollutant dispersion in the atmosphere, *Atmos. Res., 92,2009, 1-17*.
[19] Moreire D. M., Vilhena M. T., Buske D., and Tirabassi T., The GILTT solution of the advection-diffusion equation for an inhomogenous and nonstationart PBL, *Atmos. Environ., 40, 2006, 3186-3194.*
[20] Jaiswal, D.K., Kumar A., Kumar N. and Singh M. K., Solute Transport along Temporally and Spatially Dependent flows through Horizontal Semi-infinte Media: Dispersion being proportional tosquare of velocity, *Journal of Hydrologic Engineering (ASCE). 16(3) 2011, 228-238.*
[21] Jaiswal, D.K., Kumar A., Kumar N., Yadav R. R., Analytical solutions for temporally and spatially dependent solute dispersion of pulse type input concentration in one-dimensional semi-infinite media, *Journal of Hydro-environmental Research 2, 2009, 254-263.*
[22] Yadav R. R., Jaiswal D. K., Yadav H. K., and Gulrana, One-dimensional temporally dependent advection-dispersion equation in porous media, Analytical solution, *Natural Resource Modeling., 23(12), 2010, 521-539.*
[23] Dong-mei, Sun, Yue-ming, Zhu and Ming-jin, Zhang, Unsteady seepage problems due to water level fluctuation, *Journal of Tian Jin University, Vol. 40(7), 2007, 779-785.*
[24] Perez Guerrero J. S., Pimentel L. C. G., Skaggs T. H. and Van Genuchten M. th., Analytical solution of the advection-diffusion transport equation using a change-of-variable and integral transform technique, *International Journal of heat and Mass Transfer, 52, 2009, 3297-3304.*
[25] Chen J. S., and Liu C. W., Generalized analytical solution for advection-dispersion equation in finite spatial domain with arbitrary time-dependent inlet boundary

condition, *Hydro. Earth Syst. Sci., 8, 2011, 4099-4120.*

[26] Scheidegger A. E., Statistical hydrodynamics in porous media: *Appl. Phys. Jour., 25, 1054, 994-1001.*

[27] Day P. R., Dispersion of a moving salt-water boundary advancing through saturated sand: *Am. Geophys. Union Trans., 37, 1956, 595-601.*

[28] deJony G. de J., Longitudinal and transverse diffusion in granular deposits: *Am. Geophys. Union Trans., 39, 1958, 67-74.*

[29] Bear J., Dynamics of Fluids in Porous Media, *Dover Publications, New York, 1972.*

[30] Lapidus L. and Amundson N. R., Mathematics of adsorption in beds, VI The effects of longitudinal diffusion in ion-exchange and chromatographic columns, *J. Phy, Chem., Vol. 56, 1952, 984-988.*

[31] Liao S. J., Beyond Perturbation: introduction to Homotopy Analysis Method. *CRC Press: Chapman & Hall; 2003.*

[32] Liao S. J., A kind of approximate solution technique which does not depend upon small parameters (II): an application in fluid mechanics, *Int. J. Nonlinear. Mech, 1997, 32-815.*

[33] Liao S. J., An explicit, totally analytic approximation of Blasius viscous flow problems. *Int. J. Nonlinear. Mech. Vol. 34, 1999, 759–778.*

[34] Liao S. J., Magyari E., Exponentially decaying boundary layers as limiting cases of families of algebraically decaying ones, *ZAMP, 57(5), 2006, 777-92.*